Free Vibration Analysis of Functionally Graded Carbon Nanotube-Reinforced Composite Cylindrical Panels

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***Abstract*—This paper presents a free vibration analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) cylindrical panels using the element-free *kp*-Ritz method. Carbon nanotubes are assumed uniaxially aligned in axial direction of cylindrical panels and functionally graded in thickness direction with different types of distributions. The effective material properties of FG-CNTRC cylindrical panels are estimated through a micromechanical model based on the Eshelby-Mori-Tanaka approach. The governing equations are based on the first order shear deformation shell theory and the two-dimensional displacement fields are approximated by mesh-free kernel particle functions. Convergence and comparison studies have been carried out to verify the stability and accuracy of the present method for analysis of free vibration of FG-CNTRC cylindrical panels.**

***Index Terms*—free vibration, carbon nanotube, composite, panel, element-free *kp*-ritz method**

I. Introduction

Carbon nanotubes (CNTs), known as a advanced material with high strength and stiffness with high aspect ratio and low density, has drawn considerable attention from researchers in many engineering fields. With the introduction of CNTs into a polymer matrix, the mechanical, electrical and thermal properties of the resulting composites may be greatly improved. Tensile tests on composite films have demonstrated that reinforcement with only 1 wt% nanotubes results in 36-42% increase in elastic modulus and 25% increase in breaking stress [1]. Bokobza [2] reported a review about carbon nanotues and their composites and found that with the addition of 1phr (part per hundred parts of resin) of multi-walled CNTs in a styrene-butadiene copolymer, a 45% increase in modulus and a 70% increase in tensile length are achieved.

In actual structural applications, carbon nanotube-reinforced composites may be incorporated in the form of beam, plate or shell as structural components. It is thus of importance to explore mechanical responses of the structures made of CNTRC. Wuite and Adali [3] presented a multiscale analysis of deflection and stress behavior of CNTRC beams. A pure bending and bending-induced buckling analysis of a nanocomposite beam was reported by Vodenitcharova and Zhang [4]. Zhu et al. [5] carried out bending and free vibration analyses of functionally graded CNTRC plates using the finite element method. By using a two-step perturbation technique, Shen [6] presented an analysis of nonlinear bending of functionally graded CNTRC plates in thermal environments. Shen and Xiang [7] examined the large amplitude vibration behavior of nanocomposite cylindrical shells in thermal environments. For nanocomposite cylindrical shells subjected to axial and pressure loads, a postbuckling analysis was conducted by Shen in [8], [9].

The main purpose of the present work is to investigate the free vibration of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) cylindrical panels. The element-free *kp*-Ritz method based on the first-order shear deformation shell theory is employed to derive the discretized governing equations. The effective material properties of FG-CNTRC cylindrical panels are estimated through a micromechanical model based on the Eshelby-Mori-Tanaka approach. Convergence and comparison studies are provided to verify the stability and accuracy of the proposed method for free vibration analysis of FG-CNTRC cylindrical panels. The effects of boundary condition, CNT volume fraction and temperature change on characteristics of the frequency are also examined in detail.

II. Theoratical Formulations

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A. Effective Material Properties

Four types of CNTRC cylindrical panels (UD, FG-O and FG-X) with length a, radius R, span angle θ_0 and thickness h are considered in this paper. The CNTs are assumed uniaxially aligned in the axial direction of the cylindrical panels and functionally graded in the thickness direction with different types of distributions.

Distributions of CNTs along the thickness direction of these four types of CNTRC cylindrical panels are assumed to be as

$$V_{CNT}(z)=\begin{cases} V_{CNT}^{*} & (\text{UD}) \\ \left(1+\dfrac{2z}{h}\right)V_{CNT}^{*} & (\text{FG-V}) \\ 2\left(1-\dfrac{2|z|}{h}\right)V_{CNT}^{*} & (\text{FG-O}) \\ 2\left(\dfrac{2|z|}{h}\right)V_{CNT}^{*} & (\text{FG-X}) \end{cases} \quad (1)$$

where

$$V_{CNT}^{*}=\frac{w_{CNT}}{w_{CNT}+\left(\rho^{CNT}/\rho^{m}\right)-\left(\rho^{CNT}/\rho^{m}\right)w_{CNT}} \quad (2)$$

where w_{CNT} is the fraction of mass of the CNTs, and ρ^m and ρ^{CNT} are densities of the matrix and CNTs, respectively.

The Eshelby-Mori-Tanaka approach, known as the equivalent inclusion-average stress method, is based on the equivalent elastic inclusion idea of Eshelby [10, 11] and the concept of average stress in the matrix due to Mori-Tanaka [12]. According to Benveniste's revision [13], the tensor of effective elastic moduli **C** of FG-CNTRC cylindrical panels is given by

$$\mathbf{C}=\mathbf{C}_m+V_{CNT}\left\langle(\mathbf{C}_{CNT}-\mathbf{C}_m)\cdot\mathbf{A}\right\rangle\cdot\left[V_m\mathbf{I}+V_{CNT}\left\langle\mathbf{A}\right\rangle\right]^{-1} \quad (3)$$

where **I** is the fourth-order unit tensor. $\mathbf{C}_m$ and $\mathbf{C}_{CNT}$ are the stiffness tensors of the matrix and CNT, respectively. It should be note that the brackets represent an average overall possible orientation of the inclusions. **A** is the dilute mechanical strain concentration tensor and is given by

$$\mathbf{A}=\left[\mathbf{I}+\mathbf{S}\cdot\mathbf{C}_m^{-1}\cdot(\mathbf{C}_{CNT}-\mathbf{C}_m)\right]^{-1} \quad (4)$$

where **S** is the fourth-order Eshelby tensor [11] which is specialized to the case of cylindrical inclusions representative of the CNTs and depends on their orientation by Mura [14].

B. Energy Functional

According to the first-order shear deformation shell theory [15], the displacement field can be expressed as

$$u(x,y,z)=u_0(x,y)+z\phi_x(x,y) \quad (5)$$

$$v(x,y,z)=v_0(x,y)+z\phi_y(x,y) \quad (6)$$

$$w(x,y,z)=w_0(x,y) \quad (7)$$

where u_0, v_0 and w_0 represent the respective translation displacements of a point at the mid-plane of the plate in x, y and z directions; ϕ_x and ϕ_y denote rotations of a transverse normal about positive y and negative x axes, respectively. The linear strain-displacement relationships are given by

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\boldsymbol{\varepsilon}_0+z\boldsymbol{\kappa},\ \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\boldsymbol{\gamma}_0 \quad (8)$$

Then, the linear constitutive relations are expressed as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{66}&0&0\\ 0&0&0&Q_{44}&0\\ 0&0&0&0&Q_{55}\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}-\begin{Bmatrix}\alpha_{11}\\ \alpha_{22}\\ 0\\ 0\\ 0\end{Bmatrix}\Delta T\right) \quad (9)$$

where

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}},\ Q_{12}=\frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} \quad (10)$$

$$Q_{66}=G_{12},\ Q_{44}=G_{23},\ Q_{55}=G_{13} \quad (11)$$

And ΔT is the temperature change with respect to a reference state. E_{11} and E_{22} are effective Young's moduli of CNTRC plates in the principal material coordinate; G_{12}, G_{13} and G_{23} are the shear moduli; and ν_{12} and ν_{21} are Poisson's ratios.

The strain energy of the panels is expressed by

$$U_{\varepsilon}=\frac{1}{2}\int_{\Omega}\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{S}\boldsymbol{\varepsilon}\,\mathrm{d}\Omega \quad (12)$$

where

$$\boldsymbol{\varepsilon}=\begin{Bmatrix}\boldsymbol{\varepsilon}_0\\ \boldsymbol{\kappa}\\ \boldsymbol{\gamma}_0\end{Bmatrix},\ \mathbf{S}=\begin{bmatrix}\mathbf{A}&\mathbf{B}&\mathbf{0}\\ \mathbf{B}&\mathbf{D}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{A}_s\end{bmatrix} \quad (13)$$

Kinetic energy of the panels for free vibration analysis is given by

$$\Theta=\frac{1}{2}\int_{\Omega}\int_{-h/2}^{h/2}\rho(z)(\dot{u}^2+\dot{v}^2+\dot{w}^2)\,\mathrm{d}z\,\mathrm{d}\Omega \quad (14)$$

Therefore, the total energy function can be expressed as

$$\Pi_s=U_{\varepsilon}-\Theta \quad (15)$$

C. Discrete System Equations

For a cylindrical panel domain discretized by a set of nodes $\mathbf{x}_I$, I=1,…, NP, approximations of displacements are expressed as

$$\mathbf{u}_0^h = \sum_{I=1}^{NP} \psi_I(\boldsymbol{x})\mathbf{u}_I e^{i\omega t} \tag{16}$$

where $\mathbf{u}_I$ is the nodal parameter associated with node I and $\psi_I(\boldsymbol{x})$ is the shape function [16], [17], respectively. Since the shape function does not have Kronecker delta property, the transformation method is employed to enforce the essential boundary conditions in this paper.

By substituting (16), into (15), and taking the variation in the energy function yields the free vibration eigen-equation

$$\left(\tilde{\mathbf{K}} - \omega^2\tilde{\mathbf{M}}\right)\tilde{\mathbf{u}} = 0 \tag{17}$$

where

$$\tilde{\mathbf{K}} = \mathbf{\Lambda}^{-1}\mathbf{K}\mathbf{\Lambda}^{-\mathrm{T}}, \tilde{\mathbf{M}} = \mathbf{\Lambda}^{-1}\bar{\mathbf{M}}\mathbf{\Lambda}^{-\mathrm{T}}, \tilde{\mathbf{u}} = \mathbf{\Lambda}^{\mathrm{T}}\mathbf{u} \tag{18}$$

$$\mathbf{K} = \mathbf{K}^b + \mathbf{K}^m + \mathbf{K}^s \tag{19}$$

in which $\mathbf{K}$ denotes the linear stiffness matrix and $\mathbf{\Lambda}$ is the transformation matrix.

Matrices $\mathbf{\Lambda}$, $\mathbf{K}^b$, $\mathbf{K}^m$, $\mathbf{K}^s$ and $\bar{\mathbf{M}}$ are given as follows:

$$\mathbf{\Lambda}_{IJ} = \psi_I(\boldsymbol{x}_J)\boldsymbol{I} \tag{20}$$

$$\mathbf{K}_{IJ}^b = \int_\Omega \left(\mathbf{B}_I^b\right)^{\mathrm{T}} \mathbf{D}\mathbf{B}_J^b \mathrm{d}\Omega \tag{21}$$

$$\mathbf{K}_{IJ}^m = \int_\Omega \left(\mathbf{B}_I^m\right)^{\mathrm{T}} \mathbf{A}\mathbf{B}_J^m \mathrm{d}\Omega + \int_\Omega \left(\mathbf{B}_I^m\right)^{\mathrm{T}} \bar{\mathbf{B}}\mathbf{B}_J^b \mathrm{d}\Omega + \int_\Omega \left(\mathbf{B}_I^b\right)^{\mathrm{T}} \bar{\mathbf{B}}\mathbf{B}_J^m \mathrm{d}\Omega \tag{22}$$

$$\mathbf{K}_{IJ}^s = \int_\Omega \left(\mathbf{B}_I^s\right)^{\mathrm{T}} \mathbf{A}^s\mathbf{B}_J^s \mathrm{d}\Omega \tag{23}$$

$$\bar{\mathbf{M}} = \int_\Omega \mathbf{G}_I^{\mathrm{T}}\bar{\mathbf{m}}\mathbf{G}_J \mathrm{d}\Omega \tag{24}$$

where

$$\mathbf{B}_I^b = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial\psi_I}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial\psi_I}{\partial y} \\ 0 & 0 & 0 & \frac{\partial\psi_I}{\partial y} & \frac{\partial\psi_I}{\partial x} \end{bmatrix} \tag{25}$$

$$\mathbf{B}_I^m = \begin{bmatrix} \frac{\partial\psi_I}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial\psi_I}{\partial y} & \frac{\psi_I}{R} & 0 & 0 \\ \frac{\partial\psi_I}{\partial y} & \frac{\partial\psi_I}{\partial x} & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$\mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & \frac{\partial\psi_I}{\partial x} & \psi_I & 0 \\ 0 & 0 & \frac{\partial\psi_I}{\partial y} & 0 & \psi_I \end{bmatrix} \tag{27}$$

$$\mathbf{G}_I = \begin{bmatrix} \psi_I & 0 & 0 & 0 & 0 \\ 0 & \psi_I & 0 & 0 & 0 \\ 0 & 0 & \psi_I & 0 & 0 \\ 0 & 0 & 0 & \psi_I & 0 \\ 0 & 0 & 0 & 0 & \psi_I \end{bmatrix} \tag{28}$$

$$\bar{\mathbf{m}} = \begin{bmatrix} I_0 & 0 & 0 & I_1 & 0 \\ 0 & I_0 & 0 & 0 & I_1 \\ 0 & 0 & I_0 & 0 & 0 \\ I_1 & 0 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & 0 & I_2 \end{bmatrix} \tag{29}$$

Matrices **A**, $\mathbf{B}^s$, **D** and $\mathbf{A}^s$ can be calculated using either an analytical or a numerical method. Matrices $\mathbf{K}^b$ and $\mathbf{K}^m$ are evaluated with a 4×4 Gauss integration and shear stiffness $\mathbf{K}^s$ is obtained by a one-point Gauss integration. I_0, I_1 and I_2 are normal, coupled normal-rotary and rotary inertial coefficients, which are defined by

$$\left(I_0, I_1, I_2\right) = \int_{-2/h}^{h/2} \rho(z)(1, z, z^2)\mathrm{d}z \tag{30}$$

III. Numerical Results

In this section, several numerical examples are presented to explicate free vibration characteristics of FG-CNTRC cylindrical panels. PmPV [18] is selected for the matrix. As material properties of SWCNTs are charity-, size- and temperature-dependent, typical values are taken from analytical results of Popov et al. [19]. In this paper, properties of the matrix and CNTs are given as at temperature T = 300 K (room temperature) unless otherwise specified.

A. Convergence Study

To verify the present formulation, a convergence study is carried out for free vibration analysis of a clamped cylindrical panel in terms of the number of nodes. The geometry properties of the panel are length L = 76.2 mm, radius R = 762 mm, thickness h = 0.33 mm and the span angle θ = 7.64°. The Young's modulus is E = 6.8948 × 10^{10} N/m², the Poisson ratio is v = 0.33 and the mass density is ρ = 2657.3 kg/m³.

Table I lists the first six frequencies with different number of nodes. It can be seen that when panels are represented by 17×17 nodes, the result obtained by the present element-free method agrees well with solution of Au and Cheung [20]. According to its effectiveness and efficiency, discretization with 17×17 nodes has been used for all further analyses.

TABLE I. Convergency Study

Mode	Node number				Au and Cheung [20]
	11×11	13×13	15×15	17×17	
1	881	874	869	867	869
2	939	944	951	956	957
3	1310	1300	1293	1291	1287
4	1387	1375	1367	1362	1363
5	1454	1444	1438	1435	1439
6	1714	1735	1740	1748	1751

B. Parameter Studies

Several numerical examples showing the applicability of the element-free *kp*-Ritz method on free vibration analysis of FG-CNTRC cylindrical panels are presented in this section. Here detailed parametric studies are carried out to investigate the effects of CNT volume fraction and radius on frequency characteristics of various types of CNTRC panels. The effects of boundary condition and distribution type of CNTs are also examined. The non-dimensional frequency parameter is defined as

$$\bar{\omega}_{mn} = \omega_{mn} \frac{b^2}{h} \sqrt{\frac{\rho^m}{E^m}} \tag{31}$$

where ω_{mn} is the natural frequency. Subscripts m and n are the number of half-waves of mode shapes in x and y directions, respectively.

Table II shows the effect of CNT volume fraction on non-dimensional frequency parameter of various types of FG-CNTRC cylindrical panels with simply supported boundary condition. It is found that the non-dimensional frequency parameter of the panels has a higher value with a larger volume fraction of CNT since the stiffness is larger when the value of CNT volume fraction is higher. It is worth to note that FG-X panels have the highest value of frequency and FG-O panels have the lowest value of frequency among the plates due to reinforcements distributed close to top and bottom are more efficient than those distributed near the mid-plane for increasing the stiffness of FG-CNTRC cylindrical panels [5].

Subsequently, FG-CNTRC cylindrical panels with clamped boundary conditions are considered. Typical results are listed in Table III. Some similar effects of CNT volume fraction and distribution type of CNT are observed. Compared with results in Table II, we also find that the frequency parameters with clamped boundary condition are higher than those of simply supported boundary condition. That because the constraint of clamped boundary condition is stronger than the simply supported boundary condition.

Table IV shows the effect of radius on non-dimensional frequency parameter of various types of FG-CNTRC cylindrical panels with simply supported boundary condition. It can be seen the non-dimensional frequency parameter decreases as the radius of the panels increases.

TABLE II. FREQUENCY PARAMETERS FOR FG-CNTRC CYLINDRICAL PANELS WITH SIMPLY SUPPORTED BOUNDARY CONDITIONS

V^*_{CNT}	Mode	CNT distributions			
		UD	FG-V	FG-O	FG-X
0.11	1	24.27	24.12	22.36	26.00
	2	29.16	28.73	27.70	30.51
	3	34.17	34.14	32.61	35.64
	4	53.64	53.64	50.31	54.97
0.14	1	25.22	24.81	22.91	27.30
	2	29.92	29.29	28.14	31.57
	3	34.91	34.70	32.99	36.72
	4	54.22	54.12	52.53	55.88
0.17	1	26.11	25.46	23.42	28.49
	2	30.65	29.81	28.56	32.55
	3	35.62	35.24	33.34	37.73
	4	54.78	54.58	52.76	56.76

TABLE III. FREQUENCY PARAMETERS FOR FG-CNTRC CYLINDRICAL PANELS WITH CLAMPED BOUNDARY CONDITIONS

V^*_{CNT}	Mode	CNT distributions			
		UD	FG-V	FG-O	FG-X
0.11	1	41.00	38.56	36.12	44.94
	2	46.14	43.95	41.68	49.82
	3	60.78	58.56	57.40	63.70
	4	71.25	70.07	68.02	74.11
0.14	1	43.16	40.24	37.51	47.58
	2	48.14	45.51	42.93	52.33
	3	62.46	59.89	58.45	65.84
	4	72.65	71.16	68.76	76.03
0.17	1	45.09	41.78	38.80	49.88
	2	49.95	46.95	44.09	54.53
	3	64.01	61.13	59.45	67.75
	4	73.95	72.20	69.46	77.79

TABLE IV. EFFECT OF RADIUS ON FREQUENCY PARAMETERS FOR FG-CNTRC CYLINDRICAL PANELS WITH SIMPLY SUPPORTED BOUNDARY CONDITIONS

Radius	Mode	CNT distributions			
		UD	FG-V	FG-O	FG-X
0.01	1	68.29	67.63	66.97	69.32
	2	68.95	68.58	68.22	70.00
	3	78.87	78.71	78.54	79.50
	4	81.86	80.85	79.85	82.29
0.05	1	33.85	33.31	32.04	34.52
	2	37.74	37.18	35.80	38.50
	3	48.86	48.77	48.06	49.46
	4	54.78	54.40	53.09	55.68
0.1	1	24.27	24.12	22.36	26.00
	2	29.16	28.73	27.70	30.51
	3	34.17	34.14	32.61	35.64
	4	53.64	53.64	50.31	54.97

IV. CONCLUSIONS

In this paper, the element-free kp-Ritz method has been successfully applied to the free vibration analysis of the FG-CNTRC cylindrical panels. The effective material properties of the CNTRC panels are estimated through a micromechanical model based on the Eshelby-Mori-Tanaka approach. Convergence and comparison study provided to verify the accuracy of the present mesh-free method and the results agree well with the solutions available in the literature. It is found that the CNT volume fraction, radius and the boundary condition have a pronounced effect on the natural frequency parameter of the FG-CNTRC cylindrical panels. It is worth noting that the distribution type of CNT also significantly influences the natural frequency parameter of the FG-CNTRC cylindrical panels.

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