Gear Material Selection using Complex Proportional Assessment and Additive Ratio Assessment-based Approaches: A Comparative Study

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***Abstract*—Material selection is one of the most important decisions in optimal design of any manufacturing process and product. Proper material selection plays an elementary role for a productive manufacturing system with superior product and process excellence along with cost optimization. Improper material selection frequently causes enormous cost contribution and drives an organization towards immature product failure. A proficient methodology for material selection is thus required to help the manufacturing organizations for selecting the best material for a particular application. This paper focuses on the applications of two almost unrevealed multi-criteria approaches, namely complex proportional assessment (COPRAS) and additive ratio assessment (ARAS)-based methods for solving a gear material selection problem in a given manufacturing environment. A complete list of all the prospective materials from the best to the worst is obtained, taking into account multi-conflicting material selection attributes. The ranking performance of these two methods is also compared with that of the past researchers.**

***Index Terms*—gear material selection, MCDM, COPRAS, ARAS, performance analysis**

I. Introduction

Since the beginning of the manufacturing era, materials play a fundamental role for a cost effective production system to get desired outputs with improved productivity. Material selection is an essential footstep in the process of designing any physical product and its related manufacturing process. In a methodical and proficient material selection approach, the best material is selected based on its potentiality to fulfill the manufacturing objectives. In the context of product design, the main goal of material selection is to minimize cost while meeting the product performance objectives. Systematic selection of the material for a given application begins with the study of different materials and their properties. Improper material selection frequently causes huge cost contribution and drives an organization towards immature product failure. So the manufacturing designers should identify and select suitable materials with specific functionalities in order to obtain the desired product with least cost and intended applicability. However, in the light of manufacturing scenario, selection of material for a particular product is a tedious and time-consuming task to execute, because there are number of factors that have to be carefully assessed before making the final decision. For any particular application, the most important requirement may be the material strength, but depending on the working environment and functional performance, several other factors may have to be judged concurrently. Selection of the most suitable material involves the study of a large number mechanical, thermal, electrical and physical properties with cost consideration, operating environment, production process, market value, availability of supplying sources and product performance. For mechanical design, the mechanical properties of the materials are given the top priorities. The most important mechanical properties that are usually encountered in the material selection process are strength, stiffness, toughness, hardness, density and creep resistance.

The basic principle of material selection is thus to carefully identify the application requirements, then define the foremost selection criteria and finally, alternative material choices are narrowed down by the method of elimination (screening) and amalgamation of the contradictory criteria [1]-[3]. Thus, the material selection can be regarded as a multi-criteria decision-making (MCDM) problem for which a logical and systematic material selection approach is required for identifying the best alternative. The main task lies in comparing the properties of a feasible set of alternative materials and selecting the best one out of this set. But

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while choosing a material for an engineering application, the designers usually apply trial and error methods or employ their knowledge and perception which may fail at any instance.

So for selection of materials, an efficient and organized approach, based on some strong mathematical foundation, is thus required to make sure the integration between design and manufacturing objectives. Material property data sheets should never be directly used for the ultimate selection of materials. The actual performance of a particular material under different conditions may differ from the expectations.

II. Literature Review on Materials Selection

For the selection of suitable materials for diverse manufacturing and design applications, the past materials and operational researchers have introduced and developed numerous multi-criteria approaches and systems.

Jee and Kang [4] applied technique for order preference by similarity to ideal solution (TOPSIS) method to solve a flywheel material selection problem taking into consideration several technical requirements simultaneously and also used entropy approach to evaluate the weight of the material selection attributes. Sapuan [5] developed a knowledge-based system for selection of polymeric-based composite materials. Qian and Zhao [6] used the concept of 'performance index' to select appropriate material for a given micro electromechanical system design problem. The selection procedure was based on tuning the performance uniqueness to the requirements.

Milani, Shanian, Madoliat and Nemes [7] studied the effects of different criteria transformation techniques in TOPSIS method while solving a power transmission gear material selection problem. Shanian and Savadogo [8] adopted Elimination and Et Choice Translating Reality (ELECTRE)-based outranking approach for a mass produced non-heat-treatable cylindrical cover material selection problem and validated their results with the data available in the Cambridge Engineering Selector (CES). Rao [9] employed graph theory and matrix approach (GTMA) for solving two material selection problems, i.e. for a cryogenic storage tank and for a product designed for operating in a high-temperature oxygen-rich environment and also proposed a 'material suitability index' to measure the degree by which a material could be successfully selected for the given engineering design. Milani and Shanian [10] applied ELECTRE III method to rank the best compromised candidate gear materials considering criteria trade-offs, designers' preference information, data uncertainties and incompleteness. Chan and Tong [11] presented an integrated methodology of constructing an order pair of materials and end-of-life product strategy for material selection using grey relational analysis approach. Shanian and Savadogo [12] compared compromise ranking, ELECTRE IS and ELECTRE IV for solving a highly sensitive component material selection problem involving mutually conflicting design objectives. Thakker, Jarvis, Buggy, and Sahed [13] proposed a novel approach for optimal selection of wave energy extraction impulse turbine blade material combining the Cambridge Material Selector-based method, adapted value engineering technique and TOPSIS method.

Sharif Ullah and Harib [14] presented an intelligent method to treat material selection problems where design configurations, working conditions and design-relevant information were not precisely known. Rao and Davim [15] used a logical system of material selection for a given engineering design combining TOPSIS and analytic hierarchy process (AHP) methods, and proposed a 'material selection index' to help the designers to assess and grade the feasible materials. Chatterjee, Athawale and Chakraborty [16] solved a flywheel and a sailing boat material selection problems using Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) and ELECTRE II methods, and compared their relative ranking performances. Jahan, Ismail and Sapuan [17] reviewed different quantitative procedures developed to solve material selection problems for various engineering applications. The details of those methods, including application modalities, qualities and inadequacies, were mainly addressed.

Jahan, Mustapha, Ismail, Sapuan and Bahraminasab [18] proposed a new version of VIKOR method with a novel normalization technique based on criteria target values and derived a compromise algorithm for material selection problems. Chatterjee, Athawale and Chakraborty [19] suggested resolving the material selection problems using two almost new MCDM methods, i.e. COPRAS and evaluation of mixed data (EVAMIX) methods. These two methods were used to rank the alternative materials, for which several requirements were considered simultaneously approaches.

Maity, Chatterjee and Chakraborty [20] considered an exhaustive list of 19 cutting tool materials and evaluated their performances based on ten selection criteria. COPRAS-G method was then applied to solve the cutting tool material selection problem considering grey data in the decision matrix. Chatterjee and Chakraborty [21] applied preferential ranking methods for material selection. These methods have the output of a list of best-to-worst suitable materials based on the decision criteria and their relative importance.

Chatterjee and Chakraborty [22] attempted to solve the material selection problems using COPRAS and complex proportional assessment with grey number (COPRAS-G) methods while considering different material selection criteria and their relative importance. The rankings obtained using these two methods almost corroborate with those derived by the past researchers. Girubha and Vinodh [23] used VIKOR method for material selection of electric car instrument panel under fuzzy environment. Jahan, Mustapha, Sapuan, Ismail and Bahraminasab [24] proposed an explicit and logical procedure to guide designers to determine the relative importance of material selection attributes. The proposed framework covered objective, subjective and interdependency among different material selection attributes. Athawale and

Chakraborty [25] applied ten most commonly used multi-criteria approaches for solving different material selection problems and compared their relative ranking performances. Maity and Chakraborty [26] proposed a fuzzy analytic network process (FANP)-based material selection methodology and solved a supercritical boiler material problem. Maity and Chakraborty [27] applied a fuzzy ANP-based approach to select the most appropriate materials for wind energy and wave energy extraction impulse turbine blades. Karande, Gouri and Chakraborty [28] applied utility concept and desirability function approach to solve four material selection problems in discrete manufacturing environments. Prasad and Chakraborty [29] solved some material selection problems using quality function deployment (QFD) approach while integrating the voice of the customers for a product with its technical requirements. Rai, Jha, Chatterjee, Chakraborty [30] proposed a compromise ranking method in the perspective of regret theory as a tool for solving material selection problems in manufacturing environment. Chakraborty and Chatterjee [31] considered five material selection problems and solved using VIKOR, TOPSIS and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) methods to demonstrate the effect of number of criteria on the final rankings of the material alternatives. It was observed that the choices of the best suited materials solely depend on the criterion having the maximum priority value. It was also found that among the three MCDM methods, the ranking performance of VIKOR method was the best. Chauhan and Vaish [32] applied various MCDM approaches for solving a hard coating material selection problem. TOPSIS was used for ranking the alternative materials, while material selection charts were used to select the alternative hard coating materials. Hierarchical clustering was used to classify hard coating materials under study. Pearson correlation coefficients were calculated between the materials properties under study which could be integrated with materials informatics for rapidly screening and designing materials.

From the literature review as presented above, it is well understood that numerous research works have already been carried out by the past researchers for solving manufacturing and design material selection problems using different mathematical and MCDM-based methods, but very little effort has yet been employed to compare the relative performance of various multi-criteria approaches while solving the material selection problems. In this paper, an attempt is made to balance this space while comparing the ranking performances of COPARAS and ARAS methods for solving a gear material selection problem under a given manufacturing environment. Till date, these two MCDM methods have very limited applications in the material selection domain. One example is cited to demonstrate the feasibility of these two approaches. It is observed that these two MCDM methods have very high potentials to deal with such complex manufacturing decision-making problems.

III. Complex Proportional Assessment and Additive Ratio Assessment-Based Methods

A. Complex Proportional Assessment Method

The COPRAS method assumes direct and proportional dependences of the significance and utility degree of the available alternatives under the presence of mutually conflicting criteria [33], [34], [35]. It takes into account the performance of the alternatives with respect to different criteria and also the corresponding criteria weights. This method selects the best decision considering both the ideal and the ideal-worst solutions. The COPRAS method which is used here for decision-making in manufacturing environment adopts a six stage procedure for ranking and evaluating alternatives in terms of their significance and utility degree. COPRAS method has the ability to account for both positive (beneficial) and negative (non-beneficial) criteria, which can be assessed separately within the evaluation process. The most important feature that makes COPRAS method superior to other methods is that it can be used to calculate the utility degree of alternatives indicating the extent to which one alternative is better or worse than other alternatives taken for comparison. The steps for COPRAS method are presented as below:

Step 1: Normalize the decision matrix using linear normalization procedure [34]. The purpose of normalization is to obtain dimensionless values of different criteria so that all of them can be compared.

Step 2: Determine the weighted normalized decision matrix, *D*.

$$D = [y_{ij}]_{mxn} = r_{ij} \text{ x } w_j \quad (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n) \qquad (1)$$

The sum of dimensionless weighted normalized values of each criterion is always equal to the weight for that criterion.

$$\sum_{i=1}^{m} y_{ij} = w_j \qquad (2)$$

Thus, it can be said that the weight, w_j of j^{th} criterion is proportionally distributed among all the alternatives according to their weighted normalized value, y_{ij}.

Step 3: The sums of weighted normalized values are calculated for both the beneficial and non-beneficial attributes using the following equations:

$$S_{+i} = \sum_{j=1}^{n} y_{+ij} \qquad (3)$$

$$S_{-i} = \sum_{j=1}^{n} y_{-ij} \qquad (4)$$

where y_{+ij} and y_{-ij} are the weighted normalized values for beneficial and non-beneficial attributes respectively. The greater the value of S_{+i}, the better is the alternative; and the lower the value of S_{-i}, the better is the alternative. The S_{+i} and S_{-i} values express the degree of goals attained by each alternative. In any case, the sums of 'pluses' S_{+i} and 'minuses' S_{-i} of the alternatives are always respectively equal to the sums of weights for the beneficial and non-beneficial attributes as expressed by the following equations:

$$S_{+} = \sum_{i=1}^{m} S_{+i} = \sum_{i=1}^{m}\sum_{j=1}^{n} y_{+ij} \quad (5)$$

$$S_{-} = \sum_{i=1}^{m} S_{-i} = \sum_{i=1}^{m}\sum_{j=1}^{n} y_{-ij} \quad (6)$$

Step 4: Determine the significances of the alternatives on the basis of defining the positive alternatives S_{+i} and negative alternatives S_{-i} characteristics.

Step 5: Determine the relative significances or priorities (Q_i) of the alternatives.

$$Q_i = S_{+i} + \frac{S_{-min}\sum_{i=1}^{m} S_{-i}}{S_{-i}\sum_{i=1}^{m}(S_{-min}/S_{-i})} \quad (i = 1,2,\ldots,m) \quad (7)$$

where S_{-min} is the minimum value of S_{-i}. The greater the value of Q_i, the higher is the priority of the alternative. The relative significance value of an alternative shows the degree of satisfaction attained by that alternative. The alternative with the highest relative significance value (Q_{max}) is the best choice among the candidate alternatives. Step 6: Calculate the quantitative utility (U_i) for i^{th} alternative. The degree of an alternative's utility which leads to a complete ranking of the candidate alternatives is determined by comparing the priorities of all the alternatives with the most efficient one and can be denoted as below:

$$U_i = \left[\frac{Q_i}{Q_{max}}\right] x100\% \quad (8)$$

where Q_{max} is the maximum relative significance value. These utility values of the alternatives range from 0% to 100%.

Thus, this approach allows for evaluating the direct and proportional dependence of significance and utility degree of the considered alternatives in a decision-making problem having multiple criteria, their weights and performance values of the alternatives with respect to all the criteria.

B. *Additive Ratio Assessment Method*

ARAS method is based on quantitative measurements and utility theory. In this method, a utility function value determines the relative efficiency of an alternative over the other alternatives. This utility function is directly proportional to the relative effect of the criteria values and weight importance of the considered criteria. The utility value of an alternative is determined by a comparison of variant with the ideally best alternative. The steps of ARAS method are presented as follows [36, 37]:

Step 1: For the beneficial attributes, determine the normalized decision matrix using a linear normalization procedure, as proposed by Turskis and Zavadskas [37]. For non-beneficial attributes, the normalization procedure follows two steps. At first, the reciprocal of each criterion with respect to all the alternatives is taken as follows:

$$x_{ij}^{*} = \frac{1}{x_{ij}} \quad (9)$$

In the second step, the normalized values are calculated:

$$R = [r_{ij}]_{mxn} = \frac{x_{ij}^{*}}{\sum_{i=1}^{m} x_{ij}^{*}} \quad (10)$$

Step 2: Determine the weighted normalized decision matrix, D, using (1). This step is similar to COPRAS method.

Step 3: Determine the optimality function (S_i) for i^{th} alternative.

$$S_i = \sum_{j=1}^{n} y_{ij} \quad (11)$$

Higher the S_i value, the better is the alternative. The optimality function S_i has a direct and proportional relationship with the values in the decision matrix and criteria weights.

Step 4: Calculate the degree of utility (U_i) for each alternative.

It is determined by a comparison of the variant with the most efficient one (S_0). The equation used for calculation of the utility degree (U_i) is given as below:

$$U_i = \frac{S_i}{S_0} \quad (12)$$

The utility values of the alternatives range from 0% to 100%. The alternative with the highest utility value (U_{max}) is the best choice among the candidate alternatives.

IV. Illustrative Example

To reveal the computational precision and expediency of the complex proportional assessment and additive ratio assessment-based MCDM methods, a power transmission gear material selection problem from Milani, Shanian, Madoliat and Nemes [7] is considered.

Gears are generally rotary machine wheels with cut teeth. Gears mesh together and make things turn. Gears are generally used for reversing the direction of rotation, to increase or decrease speed of rotation and to move rotational motion to a different axis. Gears are also used to transfer motion or power from one moving part to another. If power is provided to turn one gear, that gear can turn another gear. Two or more gears working in tandem are called transmission and can produce mechanical advantage through gear ratio, and so, it may be considered as a simple machine.

For selection of the suitable gear material for any type of application, mainly three types of criteria, i.e. atomic bond strength, arrangement and packing of the atoms in solid material, and tooth failure are emphasized. Microstructure-insensitive properties (like density, elastic modulus and thermal properties) and microstructure-sensitive properties (e.g. strength, ductility, fracture toughness and hardness) are also predominant in gear material selection problems.

Keeping in view the above requirements, Milani, Shanian, Madoliat and Nemes [7] considered nine alternative gear materials and five selection attributes, i.e. surface hardness (SH) (in BHN), core hardness (CH) (in BHN), surface fatigue limit (SFL) (in N/mm^2), bending fatigue limit (BFL) (in N/mm^2) and ultimate tensile strength (UTS) (in N/mm^2) for the given power transmission gear material selection problem. Among these five criteria, core hardness is the only non-beneficial attribute where lower value is always preferred.

The quantitative data for this gear material selection problem is given in Table I. Milani, Shanian, Madoliat and Nemes [7] applied TOPSIS method for solving that gear material selection problem and adopted entropy approach to determine the criteria weights as w_{SH} = 0.172, w_{CH} = 0.005, w_{SFL} = 0.426, w_{BFL} = 0.292 and w_{UTS} = 0.102. These criteria weights are used here for the subsequent analyses. A complete list of all the possible choices from the best to the worst suitable materials is obtained using each of the preference ranking-based methods taking into account different material selection attributes simultaneously. The ranking performances of these two methods are also compared with that obtained by the past researchers.

TABLE I. QUANTITATIVE DATA FOR GEAR MATERIAL SELECTION PROBLEM [7]

Material	SH	CH	SFL	BFL	UTS
Cast iron (A_1)	200	200	330	100	380
Ductile iron (A_2)	220	220	460	360	880
S.G. iron (A_3)	240	240	550	340	845
Cast alloy steel (A_4)	270	270	630	435	590
Through hardened alloy steel (A_5)	270	270	670	540	1190
Surface hardened alloy steel (A_6)	585	240	1160	680	1580
Carburised steel (A_7)	700	315	1500	920	2300
Nitrided steel (A_8)	750	315	1250	760	1250
Through hardened carbon steel (A_9)	185	185	500	430	635

A. *COPRAS Method*

While solving this gear material selection problem using COPRAS method, the data of the decision matrix, as shown in Table I, is first transformed into dimensionless values using linear normalization procedure, so that all these criteria can be comparable. Then the corresponding weighted normalized matrix is obtained using (1), as given in Table II. Now, using (3) and (4), the sums of the weighted normalized values are estimated for both the beneficial (S_{+i}) and non-beneficial attributes (S_{-i}), as given in Table III.

Then, applying (7), the relative significance or priority value (Q_i) for each alternative gear material is computed, as shown in Table IV. This table also exhibits the value of quantitative utility (U_i) as calculated using (8) for each alternative on the basis of which a complete ranking of the alternative materials is obtained.

The candidate materials for designing the power transmission gear material selection problem are then arranged in descending order of U_i values yielding a complete ranking of the materials as $A_7 > A_8 > A_6 > A_5 > A_4 > A_3 > A_9 > A_2 > A_1$. The best choice is material A_7 (carburised steel) and the worst choice is material A_1 (cast iron).

TABLE II. WEIGHTED NORMALIZED MATRIX

Material	SH	CH	SFL	BFL	UTS
A_1	0.0101	0.0004	0.0199	0.0064	0.0040
A_2	0.0111	0.0005	0.0278	0.0230	0.0093
A_3	0.0121	0.0005	0.0332	0.0217	0.0089
A_4	0.0136	0.0006	0.0381	0.0278	0.0062
A_5	0.0136	0.0006	0.0405	0.0345	0.0126
A_6	0.0294	0.0005	0.0701	0.0435	0.0167
A_7	0.0352	0.0007	0.0906	0.0588	0.0243
A_8	0.0377	0.0007	0.0755	0.0486	0.0132
A_9	0.0093	0.0004	0.0302	0.0275	0.0067

TABLE III. SUMS OF WEIGHTED NORMALIZED VALUES

Material	S_{+i}	Value	S_{-i}	Value
A_1	S_{+1}	0.0404	S_{-1}	0.0004
A_2	S_{+2}	0.0712	S_{-2}	0.0005
A_3	S_{+3}	0.0760	S_{-3}	0.0005
A_4	S_{+4}	0.0857	S_{-4}	0.0006
A_5	S_{+5}	0.1012	S_{-5}	0.0006
A_6	S_{+6}	0.1597	S_{-6}	0.0005
A_7	S_{+7}	0.2090	S_{-7}	0.0007
A_8	S_{+8}	0.1751	S_{-8}	0.0007
A_9	S_{+9}	0.0737	S_{-9}	0.0004

TABLE IV. Q_I AND U_I VALUES FOR ALTERNATIVE MATERIALS

Material	Q_i	U_i	Rank
A_1	0.0411	19.6184	9
A_2	0.0718	34.2847	8
A_3	0.0765	36.5498	6
A_4	0.0862	41.1631	5
A_5	0.1017	48.5524	4
A_6	0.1603	76.5285	3
A_7	0.2094	100.0000	1
A_8	0.1755	83.8015	2
A_9	0.0745	35.5552	7

B. *ARAS Method*

In this method, from the normalized decision matrix, the weighted normalized matrix is first developed, as given in Table V. This step is similar to that of COPRAS method. Next, using (11), the optimality function (S_i) for each of the gear material alternative is calculated. Then, the corresponding values of the utility degree (U_i) are determined using (12) for all the alternatives. The utility degree weighs each alternative with respect to the most efficient one. These utility values offer a comprehensive ranking of the considered material alternatives. Higher the value of utility degree, better is the alternative. The values of S_i and U_i, and the ranking achieved by the materials are displayed in Table VI. It is revealed from this table that carburised steel (A_7) is the best chosen alternative, whereas cast iron (A_1) is the worst choice which exactly corroborates with that of indicated by COPRAS method.

TABLE V. WEIGHTED NORMALIZED MATRIX

Material	SH	CH	SFL	BFL	UTS
A_1	0.0101	0.0007	0.0199	0.0064	0.0040
A_2	0.0111	0.0006	0.0278	0.0230	0.0093
A_3	0.0121	0.0006	0.0332	0.0217	0.0089
A_4	0.0136	0.0005	0.0381	0.0278	0.0062
A_5	0.0136	0.0005	0.0405	0.0345	0.0126
A_6	0.0294	0.0006	0.0701	0.0435	0.0167
A_7	0.0352	0.0004	0.0906	0.0588	0.0243
A_8	0.0377	0.0004	0.0755	0.0486	0.0132
A_9	0.0093	0.0007	0.0302	0.0275	0.0067

TABLE VI. S_i AND U_i VALUES OF EACH GEAR MATERIAL ALTERNATIVE

Material	S_i	U_i	Rank
A_1	0.0411	0.1962	9
A_2	0.0718	0.3428	8
A_3	0.0765	0.3655	6
A_4	0.0862	0.4116	5
A_5	0.1017	0.4855	4
A_6	0.1603	0.7653	3
A_7	0.2094	1.0000	1
A_8	0.1755	0.8380	2
A_9	0.0745	0.3556	7

V. COMPARATIVE ANALYSIS

For comparing the relative performances of COPRAS and ARAS methods with respect to TOPSIS method as adopted by Milani, Shanian, Madoliat and Nemes [7] while solving this gear material selection problem, the following five tests are performed. The performance measures in the evaluation matrices are kept constant during all these tests. It is observed that in all these preference ranking-based methods, carburized steel (A_7) evolves out as the best choice for this gear material selection problem, although there are some deviations in the rankings of the intermediate materials due to different mathematical modeling as involved in these two approaches. Table VII summarizes the ranking preorders of the alternative gear materials as derived from the considered MCDM methods. The ranking performances of COPRAS and ARAS methods with respect to TOPSIS method are exhibited in Fig. 1.

TABLE VII. RANKING PREORDERS OBTAINED FROM DIFFERENT MCDM METHODS

Material	TOPSIS [7]	COPRAS	ARAS
A_1	9	9	9
A_2	8	8	8
A_3	7	6	6
A_4	5	5	5
A_5	4	4	4
A_6	3	3	3
A_7	1	1	1
A_8	2	2	2
A_9	6	7	7

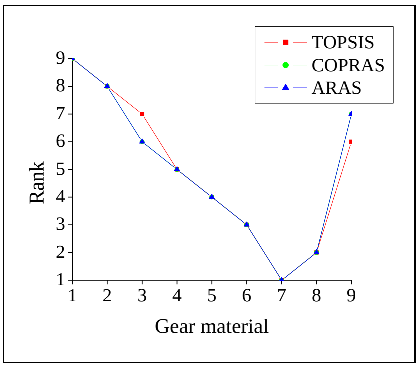
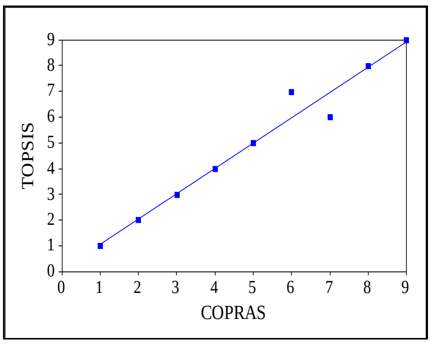
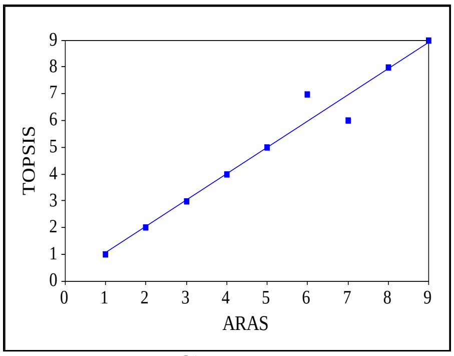


Figure 1. Comparative rankings of gear materials

- At first, scatter diagrams are plotted between the ranks obtained by Milani, Shanian, Madoliat and Nemes [7] using TOPSIS method and those derived using different preference ranking-based methods to clearly visualize the ranking similarities between them. These scatter diagrams are shown in Fig. 2 (a) and (b). A closer look at Fig. 2 (a) and (b) reveals that COPRAS and ARAS methods have produced exactly the same ranking of the alternative gear materials.



(a) TOPSIS - COPRAS



(b) TOPSIS - ARAS

Figure 2. Scatter diagrams between TOPSIS, COPRAS and ARAS methods

- In the second test, the overall ranking agreement among all these methods is determined using the Kendall's coefficient of concordance (z) value. For this gear material selection problem, z value is obtained as 0.9925, indicating an almost perfect conformity between these methods.
- In the third test, the Spearman's rank correlation coefficient (r_s) values are computed to measure the association between the ranks obtained by the various MCDM methods. From Table 8, it is observed that r_s value ranges from 0.98 to 1.00, and COPRAS and ARAS methods have perfect agreement between themselves and almost a perfect agreement with respect to TOPSIS method as adopted by Milani, Shanian, Madoliat and Nemes [7].
- The fourth test is based on the agreement between the top three ranked gear material alternatives as indicated by these methods. Here, a result of (1,2,3) means the first, second and third ranks match. Table 8 again exhibits the results of this test which indicates that all the three methods have produced exactly the same ranking preorders for the top three ranked gear material alternatives.
- Now the last test is performed to determine the overall percentage of ranks matched for these methods. It is again observed from Table VIII that COPRAS and ARAS methods show very higher percentage (77.77%) of rank matches with respect to TOPSIS method.

TABLE VIII. PERFORMANCE TEST TABLE FOR RATIO ANALYSIS-BASED METHODS

Method	COPRAS	ARAS
TOPSIS [59]	0.98 (1,2,3), 77.77	0.98 (1,2,3), 77.77
COPRAS		1 (1,2,3), 100.00

VI. CONCLUSION AND DISCUSSION

The illustrative example proves the application expediency and accuracy of COPRAS and ARAS methods while solving a complex gear material selection problem. The decision maker can easily apply these methods to evaluate the alternatives and select the most suitable material, while being completely unaware of the physical meaning of the decision-making process. The ranking preorders as derived using these two methods almost perfectly match with those as obtained by the past researchers. While applying COPRAS and ARAS methods to decision-making problems, a simple weighted summation technique is adopted separately for the normalized beneficial and non-beneficial attributes, leading to the calculation of an overall significance or utility of the considered alternatives. The main difference between the operational procedures of COPRAS and ARAS methods lies in the way they normalize the decision matrix. In COPRAS, a straightforward linear normalization is adopted, whereas, in ARAS method, a two step linear normalization technique is used. Both these two methods are relatively flexible and easy to understand, also segregates the subjective part of the decision-making process into criteria weights including decision makers' preferences.

Both the two methods can be proficiently used to any type of industrial material selection problems involving any number of qualitative and quantitative criteria, and any number of decision alternatives.

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